Int. J. Contemp. Math. Sciences, Vol. 7, 2012, no. 24, 1185 – 1192

A Concrete Application of

Adomian Decomposition Method

Roya Montazeri

Payam noor University, Iran
Montazer.roy@gmail.com

Abstract

The well known Adomian decomposition method is applied to solve a System of partial differential equations. The method is modified to be enabled to solve Systems of functional equations. Solutions resulted via Adomian Decomposition method reveal the efficiency, and the simplicity of the method.

Keywords: Adomian decomposition method ; system of differential equations

1 Introduction

Adomian decomposition method is known as a powerful device for solving many functional equations. To illustrate the method let's consider the following general form of a functional equation, in the following general form, Functions belong to a functional spase, say S.

$$F\big(u(t)\big) = g(t) \tag{1}$$

Where F is a functional operator, g is a known function in S, and we are looking for function $u(t)$, satisfying (1). To solve the functional equation (1), by Adomian decomposition method, it must be in a special form called canonical form. Let Fcan be decomposed as the following

$$F = I + R + N. \tag{2}$$

Where I is an invertible operator, R is a linear operator, and N is a nonlinear analytic operator.Considering the decomposition (2), The functional equation (1) will be written as

$I(u) + R(u) + N(u) = g,$ (3)

$I(u) = g - R(u) - N(u).$

By using the inverse of the operator I on (3) we get;

$u(t) = I^{-1}g(t) - I^{-1}R\big(u(t)\big) - I^{-1}N(u(t).$

Or

$u(t) = f(t) + L(u(t)) + G(u(t)).$ (4)

Where $f(t) = I^{-1}g(t)$ is in S, $L(t) = -I^{-1}R(t\,)$is a linear operator, and $G(u(t)) = I^{-1}g(t)$ is a nonlinear analytic operator.

The solution of this equation is supposed to be as the summation of a series, say; $\sum_{i=0}^{\infty} u_i$, which its terms will be computed recursively. It is also assumed that $G\,(u)$ is the summation of the series $\sum_{i=0}^{\infty} A_i\,(u_0, u_1 \dots)$, where $A_i'\,s$ called AdomianPolynomials which are functions of $u_0, u_1, u_2 .. u_i$ Adomian has defined these polynomials as the following

$A(u_0, u_1, \dots, u_n) = \frac{1}{n!}\left[\frac{dG(u_\lambda(t))}{d\lambda^n}\right]_{\lambda=0}.$ (5)

Where $u_{\lambda(t)} = \sum_{n=0}^{\infty} u_n\,\lambda^n,$ (6)

and λ is an auxiliary parameter.

By substitution of (6) in (4), Adomian procedure can be defined as the following

$u_0 = f(t),$

$u_1 = L(u_0) + A_0(u_0)$ (7)

$u_2 = L(u_1) + A_1\big(u_{0,}u_1\big),$

$\vdots$

$u_{n+1} = L(u_n) + A_n(u_0, u_1, \cdots, u_n).$

It has been shown that the Adomian decomposition method is quite efficient for solving many functional equations.

The governing equations to the problem are as follows:

$$-\frac{\partial^3 \mathrm{u}}{\partial \mathrm{x}^3}(\mathrm{x},\mathrm{t}) - \frac{9}{2}\frac{\partial v}{\partial x}(x,t)\frac{\partial^2 v}{\partial x^2}(x,t) + 6u(x,t)v(x,t)\frac{\partial v}{\partial x}(x,t)$$
$$+6u(x,t)\frac{\partial u}{\partial x}(x,t) - \frac{3}{2}v(x,t)\frac{\partial^3 v}{\partial x^3}(x,t) + \frac{3}{2}\frac{\partial u}{\partial x}(x,t)v^2(x,t) = 0$$
$$-\frac{\partial^3 v}{\partial x^3}(x,t) + 6\frac{\partial u}{\partial x}(x,t)v(x,t) + 6u(x,t)\frac{\partial v}{\partial x}(x,t) + \frac{15}{2}\frac{\partial v}{\partial x}(x,t)v^2(x,t) = 0 \quad (8)$$

$u_o = \frac{1}{4}c_2 - \frac{1}{4}{b_o}^2 - \frac{1}{2}b_0\sqrt{c_2}\,sech(\sqrt{c_2}\,x) - \frac{3}{4}c_2\,\mathrm{sech}(\sqrt{c_2}\,x)^2$

$v_0 = b_0 + \sqrt{c_2}\,sech(\sqrt{c_2}\,x)$

With the exact solutions:

$$u(x,t) = \frac{1}{4}c_2 - \frac{1}{4}{b_o}^2 - \frac{1}{2}b_0\sqrt{c_2}\,sech\left(\sqrt{c_2}\,\left(x + \frac{1}{2}(6{b^2}_0 + c_2)t\right)\right)$$
$$-\frac{3}{4}c_2 sech\left(\sqrt{c_2}\,\left(x + \frac{1}{2}(6{b^2}_0 + c_2)t\right)\right)^2$$

$$v(x,t) = b_0 + \sqrt{c_2}\,sech\left(\sqrt{c_2}\left(x + \frac{1}{2}(6{b^2}_0 + c_2)t\right)\right)$$

2. Solution of the system (8) by ADM.

Adomian procedure (7), for solving a functional equation says the solution is determined as soon as Adomian polynomials are calculated. This approach can be easily improved to solve a system of partial differential equations. To solve the system of differential equations (8), Adomian polynomials should be calculated. For the sake of the simplicity, let's consider the following system of partial differential equation

$$\begin{cases} u(x,t) = f(u,v), \\ v(x,t) = g(u,v). \end{cases}$$

Following the algorithm suggested in [4], let's consider an auxiliary parameter λ, and $u_\lambda = \sum_{n=0}^{\infty} u_n \lambda^n$, $v_\lambda = \sum_{n=0}^{\infty} v_n \lambda^n$ (9)

$f_\lambda(u,v) = \sum_{n=0}^{\infty} A_n \lambda^n$, $g_\lambda(u,v) = \sum_{n=0}^{\infty} B_n \lambda^n$, (10)

Where$A_n(u_0,\dots,u_n,v_0,\dots,v_n), B_n(u_0,\dots,u_n,v_0,\dots,v_n)$are Adomian polynomials. Substitution of u_λ, v_λ from (9), into $f_\lambda(u,v)$, $g_\lambda(u,v)$, (10).

Applying Taylor series expansion, trigonometric identities, and some appropriate algebraic manipulations, results in powerseries expansions of λ. Adomian polynomials will be determined by comparing the coefficients of the terms, with the same powers of λ.

$$f_\lambda(u,v) = -\frac{\partial^3 u_0}{\partial x^3} - \frac{9}{2}\frac{\partial v_0}{\partial x}\frac{\partial^2 v_0}{\partial x^2} + 6u_0 v_0 \frac{\partial v_0}{\partial x} + 6u_0 \frac{\partial u_0}{\partial x} - \frac{3}{2} v_0 \frac{\partial^3 v_0}{\partial x^3}$$
$$+ \frac{3}{2}\frac{\partial u_0}{\partial x} v_0^2 + \left[-\frac{\partial^3 u_1}{\partial x^3} - \frac{9}{2}\left(\frac{\partial v_0}{\partial x}.\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_0}{\partial x^2}.\frac{\partial v_1}{\partial x} \right) + 6\left(u_0 v_0 \frac{\partial v_1}{\partial x} + u_0 v_1 \frac{\partial v_0}{\partial x} + \right.\right.$$
$$\left.\left. u_1 v_0 \frac{\partial v_0}{\partial x} \right) + 6(u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x}) - \frac{3}{2}(v_0 \frac{\partial^3 v_1}{\partial x^3} + v_1 \frac{\partial^3 v_0}{\partial x^3} + \frac{3}{2}\frac{\partial u_1}{\partial x} v_0^2) \right]\lambda + \cdots = A_0 +$$
$$A_1\lambda + A_2\lambda^2 + A_3\lambda^3 + \cdots$$

$$A_0 = -\frac{\partial^3 u_0}{\partial x^3} - \frac{9}{2}\frac{\partial v_0}{\partial x}\frac{\partial^2 v_0}{\partial x^2} + 6u_0 v_0 \frac{\partial v_0}{\partial x} + 6u_0 \frac{\partial u_0}{\partial x}$$
$$- \frac{3}{2} v_0 \frac{\partial^3 v_0}{\partial x^3} + \frac{3}{2}\frac{\partial u_0}{\partial x} v_0^2$$

$$A_1 = -\frac{\partial^3 u_1}{\partial x^3} - \frac{9}{2}(\frac{\partial v_0}{\partial x}.\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_0}{\partial x^2}.\frac{\partial v_1}{\partial x})$$
$$+6\left(u_0 v_0 \frac{\partial v_1}{\partial x} + u_0 v_1 \frac{\partial v_0}{\partial x} + u_1 v_0 \frac{\partial v_0}{\partial x} \right) + 6(u_0 \frac{\partial u_0}{\partial x} + u_1 \frac{\partial u_0}{\partial x})$$
$$-\frac{3}{2}(v_0 \frac{\partial^3 v_1}{\partial x^3} + v_1 \frac{\partial^3 v_0}{\partial x^3} + \frac{3}{2}\frac{\partial u_1}{\partial x} v_0^2)$$

$\vdots$

$$g_\lambda(u,v) = -\frac{\partial^3 v_0}{\partial x^3} + 6\frac{\partial u_0}{\partial x} v_0 + 6u_0 \frac{\partial v_0}{\partial x} + \frac{15}{2}\frac{\partial v_0}{\partial x} v_0^2$$
$$+\left[-\frac{\partial^3 v_1}{\partial x^3} + 6\left(\frac{\partial u_0}{\partial x} v_1 + \frac{\partial u_1}{\partial x} v_0 \right) + 6\left(u_0 \frac{\partial v_1}{\partial x} + u_1 \frac{\partial v_0}{\partial x} \right) + \frac{15}{2}\frac{\partial v_1}{\partial x} v_0^2 \right]\lambda + \cdots$$
$$= B_0 + B_1\lambda + B_2\lambda^2 + B_3\lambda^3 + \cdots$$

$$B_0 = -\frac{\partial^3 v_0}{\partial x^3} + 6\frac{\partial u_0}{\partial x} v_0 + 6u_0 \frac{\partial v_0}{\partial x} + \frac{15}{2}\frac{\partial v_0}{\partial x} v_0^2$$

$$B_1 = -\frac{\partial^3 v_1}{\partial x^3} + 6\left(\frac{\partial u_0}{\partial x} v_1 + \frac{\partial u_1}{\partial x} v_0\right) + 6\left(u_0 \frac{\partial v_1}{\partial x} + u_1 \frac{\partial v_0}{\partial x}\right) + \frac{15}{2}\frac{\partial v_1}{\partial x} v_0^2$$

$\vdots$

$$u_0(x,t) = \frac{1}{4}c_2 - \frac{1}{4}b_0^2 - \frac{1}{2}b_0\sqrt{c_2}\, sec\left(\sqrt{c_2}x\right) - \frac{3}{4}c_2 sec(\sqrt{c_2}x)^2$$

$$u_1(x,t) = \int_0^t A_0\, dt$$
$$= \frac{1}{4}\frac{1}{\cosh(\sqrt{c_2}x)^3}\left(sinh\left(\sqrt{c_2}x\right)tc_2\left(b_0c_2\, cosh\left(\sqrt{c_2}x\right) + 3c_2^{\frac{3}{2}} + 3b_0^3\, cosh\left(\sqrt{c_2}x\right) + 9\sqrt{c_2}b_0^2\right)\right)$$

$\vdots$

$$u(x,t) \cong u_0(x,t) + u_1(x,t)$$
$$= \frac{1}{4}c_2 - \frac{1}{4}b_0^2 - \frac{1}{2}b_0\sqrt{c_2}\, sech\left(\sqrt{c_2}x\right) - \frac{3}{4}c_2\, sech\left(\sqrt{c_2}x\right)^2$$
$$+ \frac{1}{4}\frac{1}{\cosh(\sqrt{c_2}x)^3}\left(sinh\left(\sqrt{c_2}x\right)tc_2\left(b_0c_2\, cosh\left(\sqrt{c_2}x\right) + 3c_2^{\frac{3}{2}} + 3b_0^3\, cosh\left(\sqrt{c_2}x\right) + 9\sqrt{c_2}b_0^2\right)\right)$$

$$v_0(x,t) = b_0 + \sqrt{c_2}\,\mathrm{sech}\left(\sqrt{c_2}x\right)$$

$$v_1(x,t) = \int_0^t B_0\, dt = -\frac{1}{2}\frac{sinh(\sqrt{c_2}x)c_2(6b_0^2 + c_2)t}{cosh(\sqrt{c_2}x)^2}$$

$$v(x,t) \cong v_0(x,t) + v_1(x,t) = b_0 + \sqrt{c_2}\,\mathrm{sech}\left(\sqrt{c_2}x\right) - \frac{1}{2}\frac{sinh(\sqrt{c_2}x)c_2(6b_0^2 + c_2)t}{cosh(\sqrt{c_2}x)^2}$$

Solutions of the system are considered by two terms approximation
Solutions are plotted in figures .
Plots 1, 2, and 3, 4 show the behavior of $u(x,t)$, and $v(x,t)$, respectively.

Conclusion

Adomian decomposition method has been employed to solve a system of partial differential equations. Solutions of $u(x,t)$ from Adomian decomposition method are plotted in figure 1, and exact solution for $u(x,t)$,
Is plotted in figure 3. Also Plots of 2 and 4 show the approximate and exact solutions, $v(x,t)$,.respectively.these figures confirm the ability and the reability of the method.

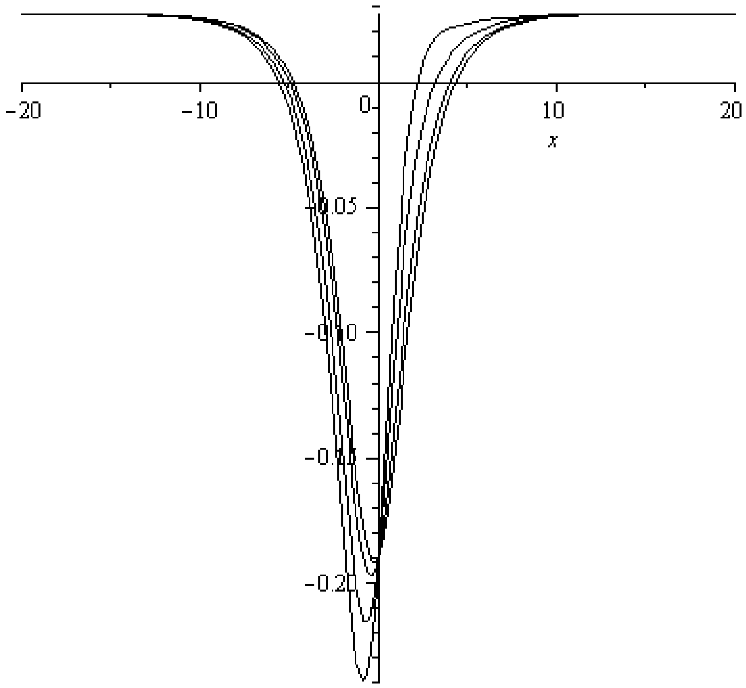

Fig. 1: plot of approximate solution,u(x,t),by ADM.

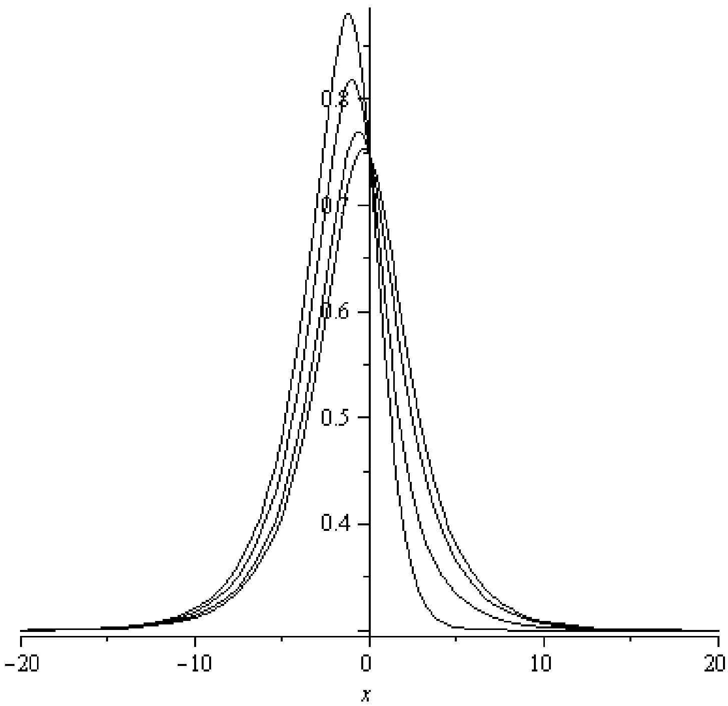

Fig. 2: plot of approximate solution ,v(x,t), by ADM.

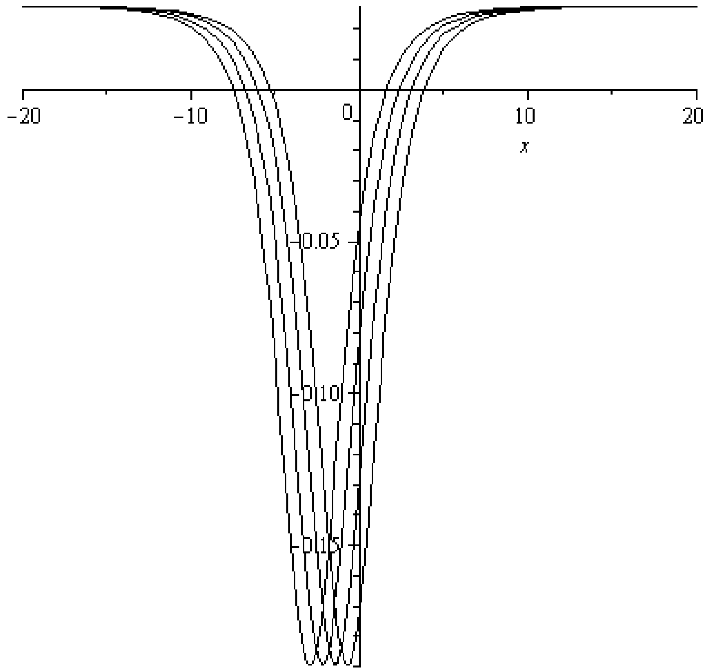

Fig. 3: plot of an exact solution ,u(x,t).

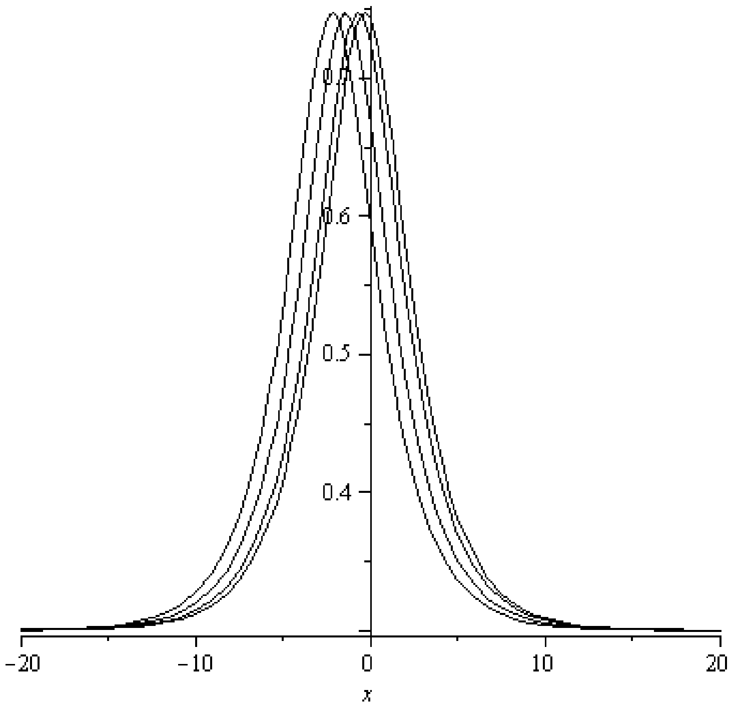

Fig. 4: plot of an exact solution , v(x,t).

References

[1] Afrouzi, G.A., Khademloo, S.: International Journal of Nonlinear Science. 2, 11−15 (2006).

[2] Cherrault,Y.: Convergence of Adomian’s decomposition method, Math. Comp. Model. 14, 83−86 (1990).

[3] Dogan Kaya: Explicit solution of Generalized Non-linear Boussinesq quations. Journal of Applied Mathematics. 1, 29−37(2001).

[4] J.Biaza, E.Babolian ,G.Kember,A.Nouri,R.Islam,An alternate algorithm for computing Adomian polynomials, App. Math. And Computations, 138/2-3 pp 523-529 (2003).

[5] J.Biazar, E.Babolian, R.Islam, Solution of systems of ordinary differential equations with Adomian decomposition method, Apple. Math. And Computation,147, PP.713-719 (2004).

Received: November, 2011